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## State of the art on the problem of vehicle routing and solid waste collection

Estado del arte sobre el problema de ruta vehículo y recogida de residuos sólidos

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### Resumen


El problema de la recolección de residuos sólidos es uno de los principales inconvenientes presentes en cualquier asentamiento humano, el cual consta de dos procesos: el primero, la recolección de residuos, y el último, la entrega de los residuos a las entidades recicladoras o puntos de destino final. El modelado de este tipo de problema de recolección puede abordarse desde la perspectiva de los modelos VRP, dependiendo de las restricciones presentes en cada área geográfica donde se aborda. Así, esta investigación aplicada a la problemática de los centros urbanos, propone un análisis del estado del arte, los diferentes problemas de recorrido relacionados y los aportes más recientes relacionados con la recolección de residuos sólidos.

*Palabras clave:* VRP, VRPPD, VRPB, Recogida de residuos, ARP

### Abstract

The problem of solid waste collection is one of the main inconveniences present in any human settlement, which consists of two processes: the first one, waste collection, and the last one, the delivery of waste to recycling entities or final destination points. The modeling of this type of collection problem can be approached from the perspective of VRP models, depending on the constraints present in each geographic area where it is addressed. Thus, this research applied to the problem of urban centers, proposes an analysis of the state of the art, the different related routing problems and the most recent contributions related to solid waste collection.

*Keywords:* VRP, VRPPD, VRPB, Waste collection, ARP

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## INTRODUCTION

Waste collection routing models can be mainly classified into two types; the first of them, based on Arcs, and the second, based on Nodes. The main difference between them lies in which element of the network will be responsible for carrying the value of customer demand. In this way, the ARP and VRP models arise, respectively.

Since this research is the prelude to a project based on a VRP model to solve a waste collection problem classified as CVRPPD, the state of the art will be more oriented towards VRP models, without losing sight of their ARP counterpart models.

In this problem, there is a set of restrictions that prevail throughout the system, such as a heterogeneous fleet of vehicles with different load capacities, nodes with and without service window periods, coupled with the uncertainty of collapse events of nodes and edges caused by demonstrations, road closures, unexpected closure of service nodes for product recycling, among others. These characteristics frame the problem as a Vehicle Routing Problem (VRP).

The following research will show the different solution techniques, as well as a general description of the problem.

### Routing problems

In order to address the problem of collecting, it is necessary to mention that there are two possible approaches that can be used when modeling this type of problem. The first of them, the models based on the Arc Routing Problem or ARP; and the second, the Vehicle Routing Problems or VRP.

ARPs can be defined as a network  $G=(V,E)$ ; where the set  $V=\{1,2,3,\dots,n\}$  is the entire set of nodes that make up the network; and the set  $E=\{e_1,e_2,e_3,\dots,e_m\}$  the set of all the arcs; in such a way that  $e_{ij}$  represents a path between a node  $i$  and a node  $j$ , where  $(i,j)\in V$ . In turn, to each arch  $e\in E$  it can be assigned a demand value  $d_e\geq 0$ . Similarly, to each arch  $e\in E$ , a cost can be added.  $c_e(e)$ , for everything  $e\in E$ . There is a one-to-one set of vehicles  $K=\{1,2,3,\dots,k\}$  with a specific capacity  $Q$  which should not be exceeded. The objective of this type of model will be to meet the total demand required by each of the arches, at minimum cost.

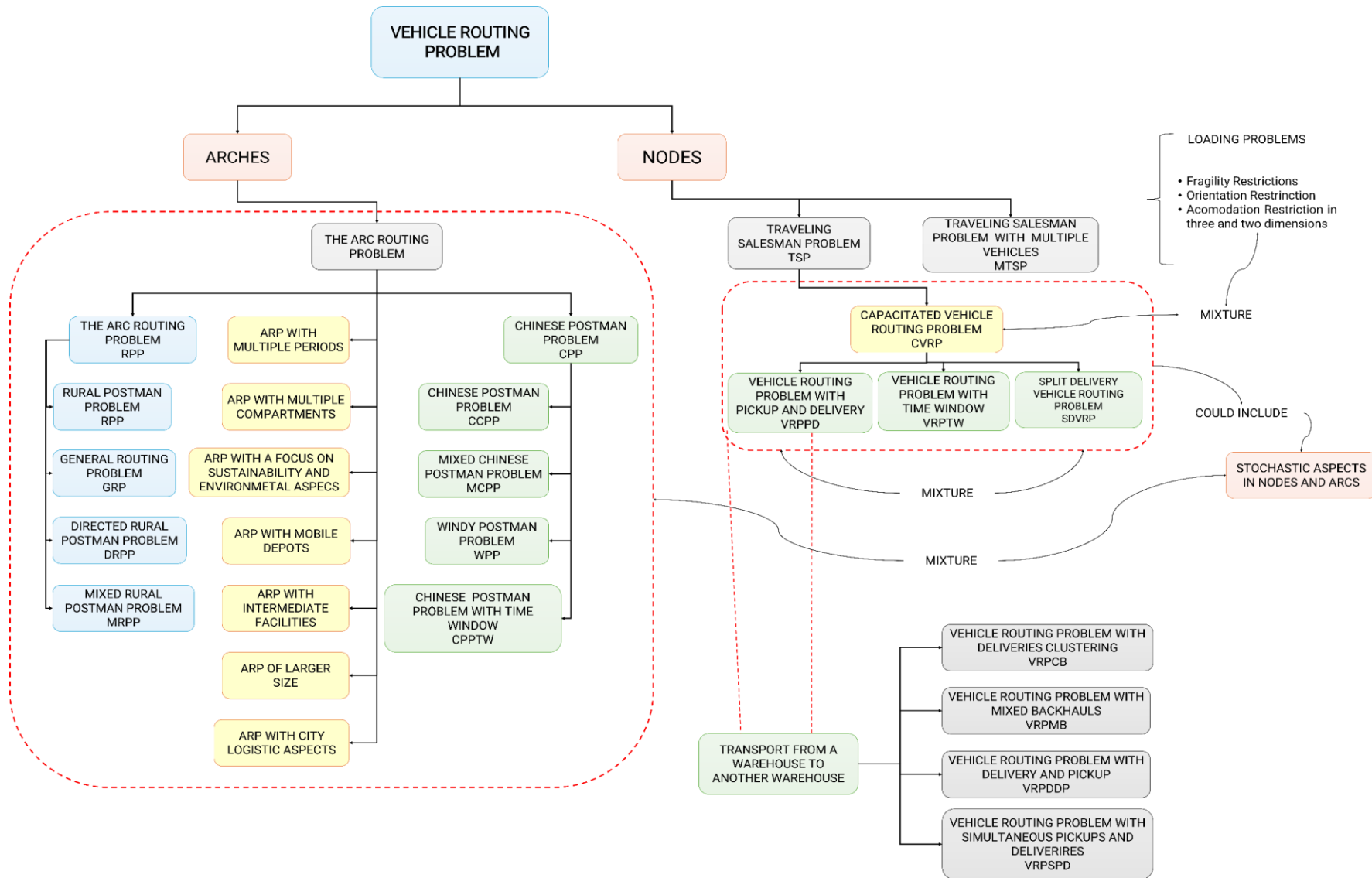
Just like ARP, VRP models can also be defined as a network  $G = (V, E)$ ; where the set  $V = \{1,2,3, \dots, n\} \cup \{0\}$  is the entire set of vertices that make up the network and the vertex  $\{0\}$  represents the Central Depot; and the set of edges  $E = \{e_{12}, e_{13}, \dots, e_{ij}\}$ , where the edge  $e_{ij}$  represents the union between a node  $i$  and a vertex  $j$  for all  $(i, j) \in V, i \neq j$ . To each arc  $e_{ij} \in E$ , a cost  $c_{ij}$  is assigned which represents the distance between a node  $i$  and a node  $j$  belonging to  $V$ . To each node  $i \in V$  a demand value  $d_i \geq 0$  is assigned, where  $d_0 = 0$ . Similarly, there is a set of vehicles  $K$  with similar characteristics and a capacity  $Q$  which may not be exceeded, i.e.,  $\sum_{i=1}^m d_i \leq Q$ . The objective of the model will be to meet all customer demand, without exceeding the capacity of the vehicles (Laporte, 2009; Mohammed, Mohd Khanapi Abd Ghani, et al., 2017). Each route  $\tau_i^\theta$  performed by a vehicle  $k \in K$  is made up of a set of nodes  $\{v_1, v_2, v_3 \dots, v_1\}$  and starts and ends at the central warehouse, in this case identified by node  $v_1$  (Laporte, 2009; Máximo & Nascimento, 2021; Mohammed, Mohd Khanapi Abd Ghani, et al., 2017). The authors (Máximo & Nascimento, 2021) define a VRP solution, as a set of routes  $\theta = \{\tau_1^\theta, \tau_2^\theta, \dots, \tau_i^\theta\}$ , where each route  $\tau_i^\theta = \{v_1, v_2^i, \dots, v_{m_i}^i\}$  in such a way that  $m_i$  is the size of the route  $\tau_i^\theta, v_0^i = v_{m_i}^i = 0$  y  $\tau_i^\theta \cap \tau_j^\theta = \{0\}$ , where  $i \neq j$  and  $\cup_{i=1}^m \tau_i^\theta = V$ .

If these two definitions are considered, it can be seen that the main difference between a VRP and an ARP lies mainly in which element of the network certain restrictions will be assigned; for example, in a VRP, the demand can be assigned to a node, while, in an ARP, it must be located on the arc.

The classification shown in Figure 1 is developed from nodes and arcs, based on (Corberán et al., 2021; Eglese & Letchford, 2000; Policroniades & Flores de la Mota, 2016). However, in (Beliën et al., 2014), another topology can be found from the elements that make up the collection system.

Depending on the context of the system, there may be constraints that involve some combination of these base models; that is, there could be a Chinese postman model, trained with stochastic time windows, or with stochastic arcs, among many other possible combinations. Load restrictions and load arrangements can be added to these combinations, as shown in Figure 1.

Figure 1  
Topological classification of VRP and ARP problems



ARP models, like VRP, can be classified as NP-Hard models (Eglese & Letchford, 2000), indicating their high level of computational complexity required to find a good solution. This complexity has turned the interest of modelers towards more efficient solution techniques, such as heuristics, metaheuristics, or hybridization techniques, each with its advantages and disadvantages. On the other hand, exact techniques usually require a long execution time during their exploration process through a feasible region with the aim of offering an exact global solution.

Below, each of the two aspects and their relationship to the waste collection problem will be addressed in general terms.

### **ARC routing problem**

The ARP models, as indicated in (Corberán et al., 2021) showed a surge of interest starting in the 1980s, when their use in urban routing problems became popular, mainly used for street cleaning or waste or snow collection systems, where each arc is assigned a certain amount of product that must be collected as the vehicle travels through it. In other words, for example, in some countries or regions the collection system consists of collecting waste from each of the homes along a street until the demand of the arc or street has been covered.

In terms of ARP contributions, it is possible to identify the following:

In (Ghiani et al., 2005), an ARP is solved in phases; the first of them focused on the development of arc clusters. The second phase aimed at route optimization, using a heuristic algorithm developed by authors, which improves route performance by 8%. The authors (Del Pia & Filippi, 2006) developed an algorithm Variable Neighborhood Descent VND to solve a real waste collection problem in northern Italy classified as CARP. Their results showed a 20% improvement over another previously used algorithm. In (Rodrigues & Soeiro Ferreira, 2015), two methods were proposed, the first one a Mixed Capacitated Arc Routing Problem with Heterogeneous Fleet, and the second one, a Mixed Capacitated Arc Routing Problem with Limited Multi-Landfills (MCARP-LML) to resolve the real case of waste collection in Monção Portugal. In (Sgarro et al., 2024), a DCPP problem was solved by means of an ACO algorithm, obtaining behavior similar to an exact algorithm designed for the problem. The authors (Tlili et al., 2014) applied a hybrid algorithm between a TS and VNS to resolve a real case of mail delivery in the Jendouba region, northwest of Tunisia. Their results were compared with those of the company and obtained better performance, with savings between 27.47% and 31.5% on the route. In (Xin et al., 2022), a problem Time-dependent rural postman problem (TDRPP) considering policies first-in-first-out (FIFO) was solved by means of a GA algorithm, comparing its results with respect to an ACO and a VNS, obtaining better performance. The authors (J. Li et al., 2021) developed a formal definition of the Heterogeneous CPP problem.

### **The vehicle routing problem (VRP)**

Vehicle Routing models can be described as problems consisting of a set of nodes and arcs; where the main objective is to satisfy the demands of each customer by means of a set of routes assigned to one or several vehicles, with similar or heterogeneous characteristics. Each of these routes has the particular characteristic of starting and ending at the same node (Laporte, 2009; Policroniades & Flores de la Mota, 2016; Torres Matus, R., 2005).

The VRP, first introduced by Dantzig and Ramser in 1959, has evolved over time by adding multiple features and constraints that allow it to be adapted to virtually any real-life routing problem [6]. These adaptations are mainly governed by the complexity of the models and reality, which are classified as NP-Hard models within combinatorial optimization, indicating their high complexity in order to obtain a solution (Alweshah et al., 2022; Berhan et al., 2014; Gendreau et al., 1996; Mohammed, Mohd Khanapi

Abd Ghani, et al., 2017; Policroniades & Flores de la Mota, 2016; Solomon, 1987). In (Policroniades & Flores de la Mota, 2016) and (Berhan et al., 2014) it is possible to find a detailed description of the different routing models, both stochastic and deterministic, as well as of their certain variants that will not be studied in this work.

In the last decade, a new type of VRP model called VRP Rich has emerged, which mixes the traditional constraints of a VRP with other real-life constraints that give models the ability to solve problems with greater complexity and uncertainty, that are commonly faced by traditional problems (Lesch et al., 2022).

The VRP over time, since its origins, has been applied to multiple real-life and academic problems that have helped to increase its taxonomy and complexity, as mentioned in (Braekers et al., 2016). In addition to these developments, one can inherently identify the computational evolution linked to the solution models. As previously mentioned, an Exact algorithm will require a greater computational effort to exhaustively review all the possible solutions and show the one with the best characteristics.

**Table 2**

*Classification of the models analyzed*

Model Type	Source
VRPTW	(Aggarwal & Kumar, 2021; Boudali & Ragmoun, 2022; Chen & Shi, 2019; Dhahri et al., 2016; Ferreira et al., 2018; Mazzuco et al., 2018; Neves-Moreira et al., 2018; Subramanyam et al., 2018; Suppan et al., 2022; Torres-Tapia et al., 2022; Y. Wang, Wang, et al., 2019)
VRPPD	(Belgin et al., 2018; Dominguez et al., 2016; Y. Li et al., 2016; Liu & Jiang, 2022; Segura et al., 2022)
VRP With heterogeneous or homogeneous fleet	(Bevilaqua et al., 2019; Brandão, 2009; Fachini et al., 2022; Salcedo-Moncada et al., 2023; Z. Wang et al., 2015)
MDVRP	(Alinaghian & Shokouhi, 2018; Bezerra et al., 2023; Ge et al., 2023; Gianessi et al., 2016; Karadeniz et al., 2022; wan et al., 2023; Y. Wang, Assogba, et al., 2019)
CVRP	(Ahkamiraad & Wang, 2018; Barrero et al., 2022; Bettinelli et al., 2014; Bulhões et al., 2018; Chiuissi et al., 2022; Cömert et al., 2022; Damião et al., 2023; Defryn & Sörensen, 2017; Elgharably et al., 2022; Fernández Gil et al., 2023; Hokama et al., 2016; Karagul & Sahin, 2023; Poonthalir & Nadarajan, 2019; Reil et al., 2018; Revanna & Al-Nakash, 2023; Rincon-Garcia et al., 2018; Sbai et al., 2022; Tarhini et al., 2022; Yelmewad & Talawar, 2021)
IRP	(Hasni et al., 2017; Jafarian et al., 2019; Kumar & Munagekar, 2022)
VRP Cross-Docking	(Baniamerian et al., 2019; Gunawan et al., 2022)
Open VRP	(Brandão, 2018; Niu et al., 2018)
Rich VRP	(Alcaraz et al., 2019; Kramer et al., 2019; Rabbouch et al., 2021)
Stochastic Arcs	(Andreatta et al., 2016; Lombard et al., 2018; Miranda et al., 2018)
Stochastic Nodes	(Bianchi et al., 2006; Borisovsky, 2023; Mavrovouniotis et al., 2023)
Combination of the previous	(Aghighi et al., 2023; Goel et al., 2019)

Table 2 show a brief general taxonomy of the VRP, based on (Policroniades & Flores de la Mota, 2016) and (Braekers et al., 2016) including some contributions from recent years.

In (Arnold & Sörensen, 2019), They indicate that a solution can be evaluated based on two characteristics. In this way, a solution can be evaluated from two main perspectives and the combination of these two: the first one being the processing time, and the second, no less important

than the first, the quality of the solution. Generally, in real life, a good or optimal solution is usually sacrificed from the execution time of the algorithm in order to show a feasible or optimal solution; and this is understandable, as there are other intangible factors that come into play at those times, such as: On-time delivery, customer loyalty, among many others. In other words, time is money, and an algorithm whose solution is delivered in two or three hours is not a viable option for a route designer who has only a few minutes to define an entire delivery plan. This is where solution techniques such as metaheuristics or simple heuristics are presented, which are focused on reducing the computational effort, delivering the best possible solution in the shortest possible time. At present, as will be seen in the state of the art, these metaheuristic solutions can be compared satisfactorily with exact techniques.

Thus, it can be said that routing problem solving methods are mainly divided into heuristic, metaheuristic, exact, and hybrid techniques (Elshaer & Awad, 2020), where the latter may be a combination of two or more of the previously mentioned techniques. However, (Máximo & Nascimento, 2021) indicate that exact techniques require more computational effort than metaheuristic techniques, however, mechanisms that bring them closer to optimal solutions and drastically reduce processing times have been developed recently.

### **The vehicle routing problem and the solid waste collection problem**

In real urban VRP problems, networks are often exposed to unexpected events that can change operating conditions at any time. One of the proposals that has brought complexity to routing models is the use of Simulation-Optimization, which allows integrating stochastic aspects or events of higher complexity to VRP or optimization models in general, merging these two approaches, as mentioned in (Bierlaire, 2015; El-Gharably et al., 2013; Fan et al., 2009; Herazo-Padilla et al., 2015; Juan et al., 2013; Policroniades Chípuli & Flores de la Mota, 2021). Another advantage of simulation is the ability to generate different scenarios for case studies and decision making.

Linked to the VRP classification mentioned above, it is possible to find different aspects depending on the restrictions associated with the nodes, arcs or vehicles that make up a distribution network, which are mentioned in (Policroniades & Flores de la Mota, 2016) and (Mohammed, Ghani, et al., 2017). These restrictions can be of the following type: time windows of attention, vehicle capacity, type of vehicles, time periods, among many others that can be mixed as the complexity of the reality allows it.

With regard to urban VRP models, such as transportation or garbage collection systems, we have the contributions of (Boudali & Rigmoun, 2022; Chen & Shi, 2019; Hasni et al., 2017) who identify the need to integrate traffic status and route visualization by means of applications such as QGIS or Arc GIS or maps to the transport models, in order to integrate a better understanding of the reality.

The authors (Segura et al., 2022) propose a Tabu Search algorithm for pickup and delivery in the city of Trujillo, Peru. To test their algorithm, they only compare with respect to previous real instances the solution to their algorithm (Chiussi et al., 2022) applied an iterative heuristic, which they compare with mixed integer programming to solve a waste collection and delivery problem; despite not being able to approach the optimal solution, they significantly improve the results of previously solved instances of the waste collection system.

In (Mahéo et al., 2022), the authors proposed the location of collection points, managed as a kind of inventory that must be emptied periodically so that people can continue to deposit their waste in them. The authors (Minh et al., 2013) applied a Memetic algorithm by means of a multi-objective optimization where they minimize the number of vehicles and the total route time, looking for new possible routes to the waste dumps. In (Angelelli & Speranza, 2002) a Tabu Search algorithm was also used for waste collection using vehicles with removable loading areas, giving the ability to load organic, inorganic or other types of waste. Another important aspect of this study lies in the category of the VRP model as a

PVRP model (Pick-up Vehicle Routing Problem) for the collection model, and a delivery model, or DVRP (Delivery Vehicle Routing Problem) to deliver organic waste to the treatment plants. The authors (Expósito-Márquez et al., 2018) develop a GRASP algorithm for plastic and cardboard waste collection process in Spain, improving the previous collection process. In (Rodríguez et al., 2021) was developed a Hill Climber algorithm to solve a CVRP problem for garbage collection in three municipalities in Spain. As previous authors, they improved the collection system. The authors (Valizadeh et al., 2021) solves a real case of garbage collection in Iran, in the Kermanshah region, considering aspects of energy consumption, carbon dioxide emissions. For its solution, an algorithm designed in GAMS was used.

A characteristic that prevails in each and every one of these models lies mainly in the waste collection process, which is followed by a process of waste delivery to a recycling or processing center.

### Heuristic and metaheuristic techniques applied to VRP

The following is a brief analysis of the state of the art of different models directly and indirectly related to the problem under study.

In (Elshaer & Awad, 2020) and (Policroniades, G. et al., 2018) it is possible to appreciate how the most likely method to solve vehicle routing problems is usually the one that applies metaheuristic techniques and hybridizations between them, depending on the strength of each algorithm to be used, i.e. diversification or intensification. The authors (Elshaer & Awad, 2020) develop a taxonomy of the solution techniques.

In previous works, just to mention a few: in 2014, (Tlili et al., 2014) developed a hybrid algorithm between a PSO and a VNS to Solve a Vehicle Routing Problem with Capacity and Distance Constraints (DCVRP), obtaining good solutions when compared with respect to optimal results. In the year 2018, (Vaferi et al., 2018) applied a two-phase solution technique to solve a VRP problem with Picking, where for the construction stage, they use a Nearest Neighbor Search algorithm, and in the second phase a Greedy Randomized algorithm. In their results, they obtain better solutions than other metaheuristic techniques, and reduce execution times. The authors (Simsir & Ekmekci, 2019) developed an Ant Colony algorithm to solve a VRP problem with simultaneous pickup and delivery VRPSDP; in their results, it can be seen how they obtained practically the optimal values, with a percentage of deviation GAP of 1.12%. (Solano-Charris et al., 2015) solved an CVRP spread by applying costs with uncertainty, which they call VRP Robust by means of a Variable Neighborhood algorithm; they developed the mathematical model and solved it in an optimal way to compare a set of instances created for this purpose was solved. In (Alinaghian & Shokouhi, 2018), a VRP with multiple compartments was solved by hybridization of an ALNS and VNS algorithm using the VNS algorithms for the intensification process and the ALNS algorithm for the diversification process. In their results, they showed that the hybrid algorithm obtains the exact solutions compared to the separately executed each technique. The authors (Y. Wang, Assogba, et al., 2019) presented a hybridized algorithm in stages; first they use a savings algorithm with 2-opt trade-offs to obtain the initial solutions, and then they use a Particle Swarm algorithm to find the best possible solution.

In the year 2021, (Ban & Nguyen, 2021) proposed a hybridization between a (TS) algorithm, (VNS), and a GRASP to solve a routing problem with an asymmetric distance constraint. The authors (Máximo & Nascimento, 2021) developed a hybrid algorithm between an ILS and VNS to solve a CVRP and thus avoid deadlocks in local optimal, where the VSN algorithm was used for the local search process.

In the year 2022, the authors (Lesch et al., 2022) applied GA, LS and ACO algorithms to solve a VRP Rich applied to road freight transport in Germany, considering the following restrictions: Vehicle capacity, dangerousness of the load, driver fatigue, attention windows, loading speed, among others. In the end, they determined that the algorithms GA and LS got the best solutions. In (Voigt et al., 2022)

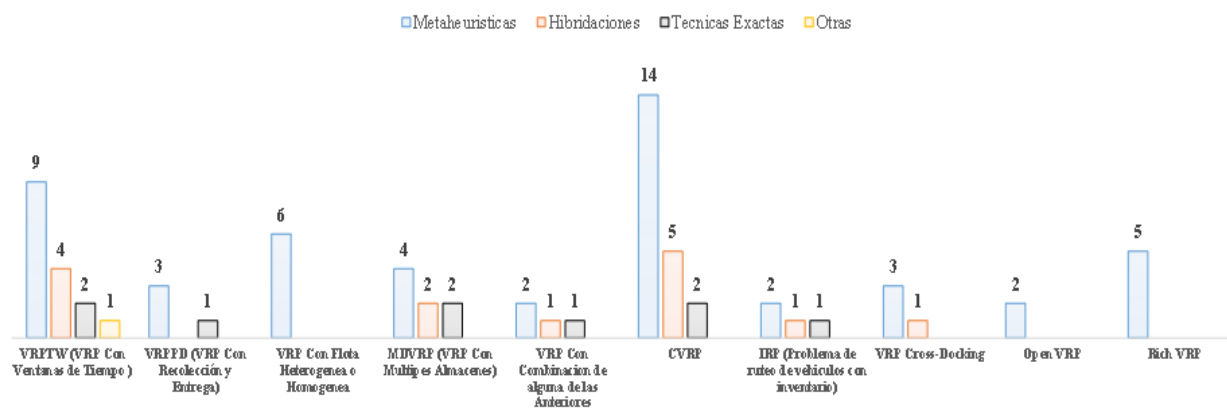
a hybrid algorithm is showed using an ALNS and a GA applied to improve the first phase of individual creation. [89] solved a VRP model with a heterogeneous fleet of pickup and delivery vehicles by means of an Adaptive Variable Neighborhood Search algorithm for 150 customers; however, their algorithm when compared to the optimal one showed a GAP of 64%. In (Salehi Sarbijan & Behnamian, 2022), it was developed a hybrid algorithm between a PSO and an SA to solve a VRP Multifreight problem. Their algorithm, in comparison with respect to an ACO, PSO and VSN, show better consistency and a lower GAP with respect to the optimal value; they propose to use the PSO algorithm for the diversification process.

In 2023, (Wu et al., 2023) resolved a VRPTW problem through an ACO algorithm and 2-opt exchanges. For the comparison process, they used a TS algorithm, noting that the ACO, in all cases of comparison, obtained a better level of performance. The authors (Rezaei et al., 2023) applied a hybridization CVRP model, between a GA algorithm and an Imperialist Competition algorithm whose mechanism is similar to a Genetic algorithm, demonstrating that this hybridization is able to compete with respect to other algorithms shown in the literature. (Becker et al., 2023) applied a modification to a Local Search algorithm using different types of exchanges in order to resolve a CVRP and compared its results with respect to existing instances; their results showed an excellent performance of the algorithm in terms of solution quality. The authors (Vieira et al., 2023) presented a solution for a VRP model with heterogeneous fleet by means of a hybrid algorithm between an Adaptive Variable Neighborhood Search and a Genetic one; their results competed with the optimal values, as good as the best-known solution reported.

Based on the small sample collected in Table 2, it is possible to develop Graph 1, which shows the large number of articles focused on solving the CVRP, compared to the other aspects of this same model. Likewise, one of the most widely used techniques for solving routing problems is usually the metaheuristic technique. The reason is due to the great computational complexity linked to this type of models. An exploration process with an Exact algorithm when the number of nodes in the network grows enormously becomes unfeasible by means of exact techniques. Other solution techniques that tend to be used to a lesser extent, are the classical heuristics that during the search process can get stuck in local optima.

**Graphic 1**

*Number of papers of different VRP models versus solution technique*

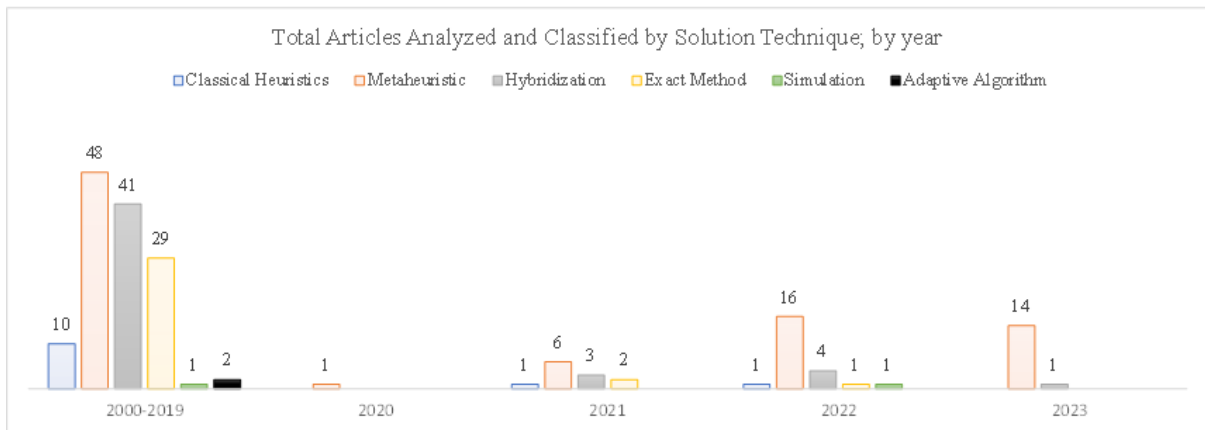
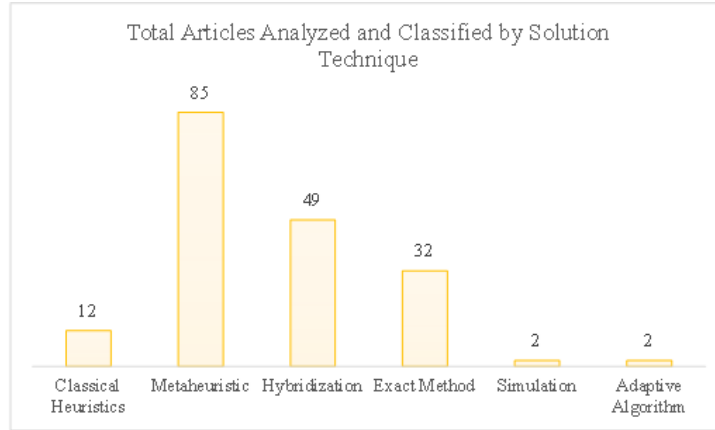


Performing an update to the tables displayed in (Policroniades, G. et al., 2018) and analyzing each one of the data based on the classification of (Brownlee, 2012) for the different metaheuristic techniques,

it is possible to obtain graphic 2, which shows the number of articles according to the solution technique used.

**Graphic 2**

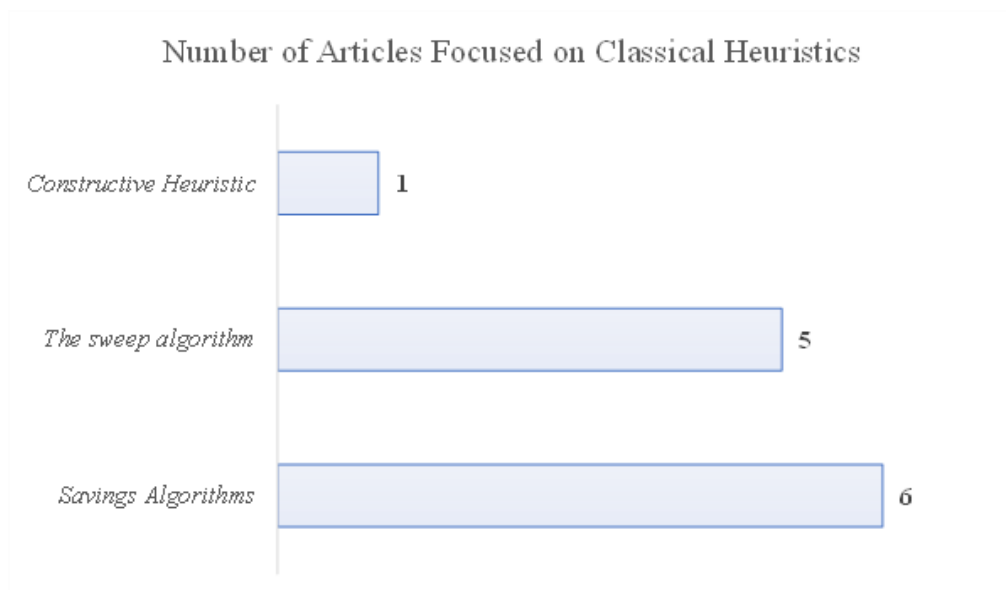
*Total articles analyzed*



It is important to note that the source is not mentioned; however, these statistics provide insight into those solution techniques that are less explored. The sample size used for this analysis was 181 articles, between 2000 and 2023, as shown in graphic 3.

**Graphic 3**

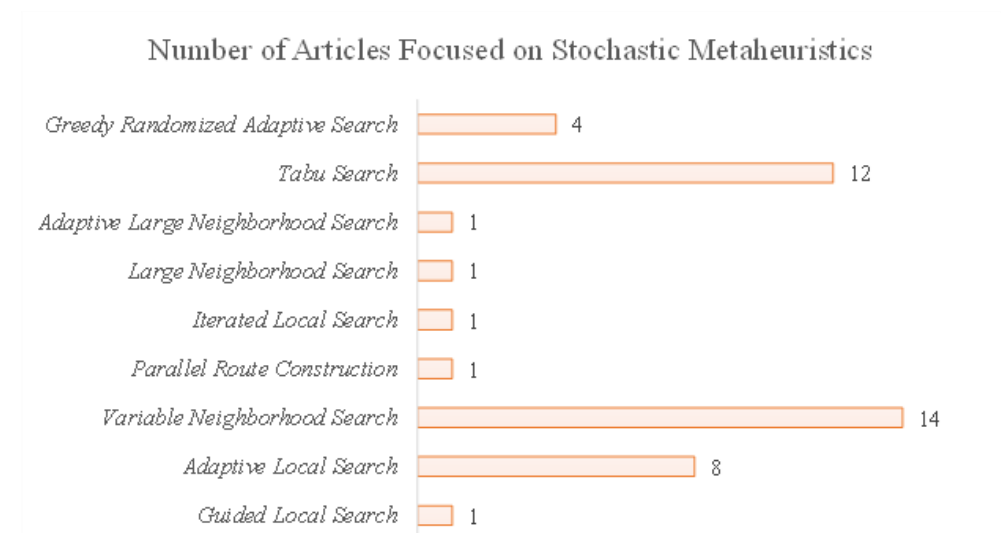
*Number of Articles Focused on Classical Heuristic*



Graphic 4 shows the statistics concerning classical heuristic techniques, in which exchange heuristics such as 2-opt or 3-opt are the most commonly used, in conjunction with savings heuristics as proposed by Carl and Wright in their solution for the VRP.

**Graphic 4**

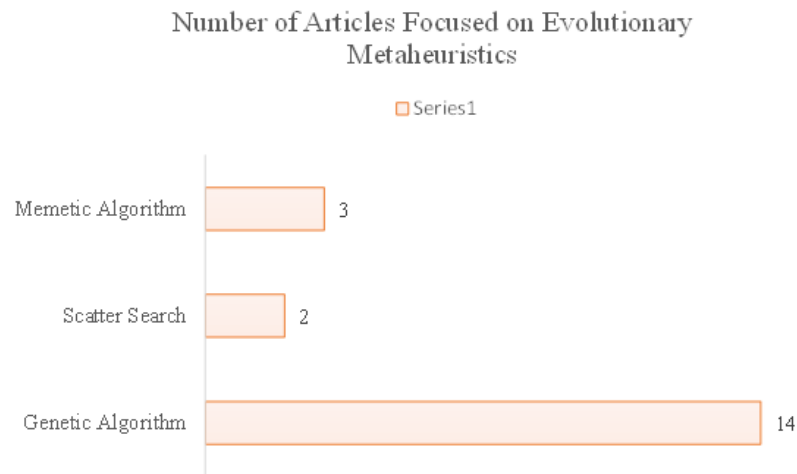
*Number of Articles Focused on Stochastic Metaheuristics*



A characteristic of metaheuristic techniques based on stochastic aspects lies in the random factor underlying their search process to provide a solution. Graph 6 shows some of the algorithms belonging to this classification, of which, among the most commonly used for solving VRP models are the Tabu Search (TS) algorithm and the Variable Neighborhood Search (VNS) algorithm. Another characteristic of these techniques is that their search process starts from a known best solution, which by means of trade-offs and specific parameters tends to converge to a global optimum.

**Graphic 5**

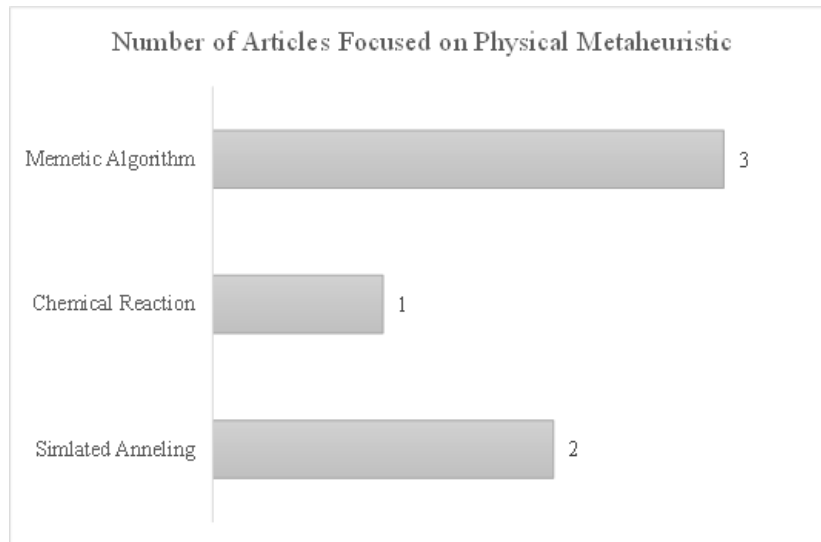
*Number of Articles Focused on Evolutionary Metaheuristics*



In terms of Evolutionary algorithms, as shown in graphic 6, they owe their origin to the concept of natural evolution, transferred to the computational concept as Evolutionary Computation, which is mentioned in (Zenil Chávez, 2005). One of the most commonly used Evolutionary algorithms to solve VRP models are Genetic algorithms. These algorithms start their search process from a random initial population which they improve by means of genetic operators such as recombination (intensification) and mutation (diversification).

**Graphic 6**

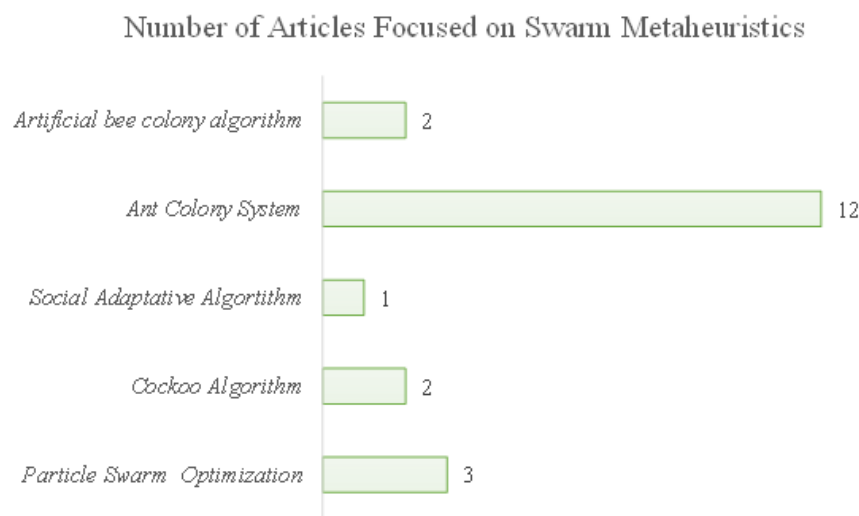
*Number of Articles Focused on Physical Metaheuristics*



Another large group of clearly known metaheuristic techniques are the metaheuristics based on physical phenomena, as shown in graphic 7. These metaheuristics, like the stochastic ones, start from a factible solution, which is analyzed and improved by specific methods until a global solution is reached. For example, in the case of simulated annealing, a heating process is emulated until it reaches a desired temperature at which the solution is expected to reach a local optimum.

**Graphic 7**

*Number of Articles Focused on Swarm Metaheuristics*

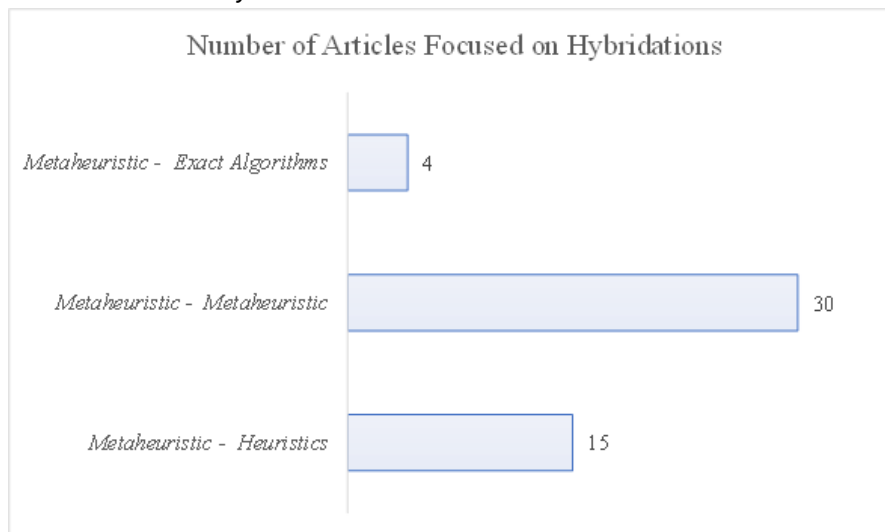


Swarm techniques have the particular characteristic of working with a random initial population of solutions; in this case, each individual in the swarm seeks to improve the solution locally by being attracted to a global solution that emerges as a consequence of the search of each individual in the swarm. In the end, the best global solution will be selected. By working with a set of individuals, the algorithm is allowed to walk away from local optima, since each of the individuals searches in a different feasible region. One of the favorite solution methods for the VRP within this group of techniques is the ACO algorithm, which is shown in graphic 9.

Last but not least, there are hybridization techniques, which seek to improve the weaknesses of one algorithm through the strengths of another. In these terms, it is very important to know whether an algorithm has a diversification or intensification strength, in order to know at what point in the search process it should be used. Hybridization between metaheuristic techniques is among the most commonly used methods. Graph 8 shows 30 articles in which this type of technique has been used.

**Graphic 8**

*Number of Articles Focused on Hybridations*



In 2012, Brownlee (2012) developed a topological classification of the VRP. Based on this classification, figures 5,6,7, and 8 are constructed. It can be seen that learning algorithms have not been highly explored. In the brief collection of articles studied, no clear reference to any application of these techniques to the solution of the VRP was identified.

The reader is encouraged to explore further to find possible contributions not identified in this paper.

### The vehicle routing problem with pickup and delivery

The VRPPD is a subclass of the VRPB models. Within the VRPB classification, there are models in which the collection can be performed in the following ways: 1). Simultaneous to the delivery process; 2). Separated from the delivery process, with the delivery process being carried out first and the pick up process following; and finally 3) where the process is similar to point 2); however, the pick-up process is carried out firstly.

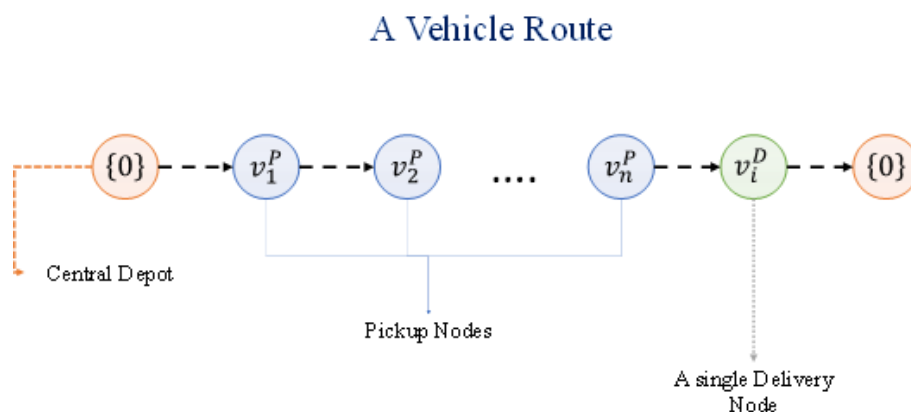
The garbage collection system shown in this state of the art proposes a separate collection and delivery process for solid waste and can therefore be classified as type 3). In (Parragh et al., 2008a) this type of problem is identified as a subclass of the VRPBs under the category of PDVRP.

In this way, the model can be defined as: undirected and complete network  $G = (V, E)$ , where  $V = V^P \cup V^D \cup \{0\}$ , being  $\{0\}$  the central warehouse understood as the whole set of nodes that make up the network  $G$ , and where  $V^P$  represents the collection nodes and  $V^D$  indicates the delivery nodes. The set  $E = E^P \cup E^{PD}$  can be visualized as the entire set of arcs linking all nodes within the network. Each of these arcs is assigned a non-negative real value  $c_{ij}$  which represents the cost of the path between a node  $i$  and node  $j$ , where  $i, j \in V$ .

A route  $r^k$  assigned to a vehicle  $k \in K$ , fulfilling the definition of a VRP, must start and end its route at the central warehouse, which makes it necessary to define a route with the following set of nodes  $V^{r^k} = V^P \cup V^D \cup \{0\}$ , where  $|V^D| = 1$  for each route  $r^k$ , and should always be located downstream of the collection nodes. This concept is shown in Figure 2.

Figure 2

Schematic representation of a route



As can be seen in figure 2, the model used for the collection and delivery of solid waste in CDMX is very similar to the VRP models with Backhauls; the only difference is during the collection process, which is carried out prior to delivery. Based on this assumption, the model can then be classified as a PDVRP.

In (Parragh et al., 2008a) and (Parragh et al., 2008b) the state of the art and classification of the VRPB models up to the year 2008 is presented. Table 2 shows some contributions from more recent years. In addition, in (Marinakis et al., 2018) different mathematical models are found for the most popular types of VRP, including VRPB and VRPPD. In (Doerner et al., 2016) it is mentioned that there is an average of one to two articles per year for VRPPD models.

In terms of the VRPPD, the following contributions were identified: (Sukhpal & Kumar, 2023) solve a VRPB problem with multiple compartments, for which they design a Branch and Bound algorithm to find near optimal solutions. The authors (Subramanian & Queiroga, 2020) propose an ILS algorithm to solve a VRP with Backhauls, proposing different solution options, among which are to first perform the collection and delivery process together, or separately, finding that the separate process generates a reduction in algorithm execution times. In (Kocat&uuml;l et al., 2021) performed a hybridization process between a VNS algorithm and one they defined as GRAMPS, using the VNS algorithm for the local search process. The authors (Queiroga et al., 2020) propose an exact Branch and Bound and Cut algorithm to solve a VRPB problem where first the delivery and then the collection process are performed, they applied their model to solve literature instances of 100 and 80 customers. In (Quila et al., 2020) a VRPBTW problem is solved by exact methods, whose model is based on integer programming for 10, 15, 20 and 30 nodes. The solution characteristic was based on splitting the delivery and collection customers.

In (Santos et al., 2023) a VRPDDP and a VRPSPD are solved, which are identified as highly relevant problems due to their application in reverse logistics. Their models involved multi-objective problems, in which apart from reducing route costs, they minimize CO2 emissions. Heuristic techniques and an Exact algorithm are applied to solve them.

The PDVRP is identified in the processes of food collection and delivery to customers, as mentioned in the following section (Gunes et al., 2010). In (Raffaele et al., 2023) a PDVRP is solved, in which deliveries and collections are carried out simultaneously; likewise, the authors applied two phases, in the first one they group customers by proximity and in the second one they solved the routing model in an exact way. The authors (Deep & Tanmay, 2022) solved a PDVRP model for a food pickup and delivery problem; the authors solve by means of Google Map, and the shortest path concept to obtain feasible routes for the routing problem, in which the delivery driver must leave a given point and return to the one after the pickup and delivery process.

## CONCLUSIONS

As can be seen, any waste collection system consists of a collection and delivery process; the latter can be to recycling centers or to final destination points. These characteristics allow us to classify this type of model generally as a Vehicle Routing with Collection and Delivery (VRPPD) model.

VRPPD models, like any VRP model, can be solved by different techniques; metaheuristic techniques stand out for their greater efficiency due to their wide advantages, based on the execution times and the quality of their solutions. Among these techniques, the VNS, TS and GA algorithms are the most popular.

Solid waste collection, in any region of the world, can be highly complex due to the size of the network representing the different collection points, coupled with routing constraints that may involve a homogeneous or heterogeneous fleet of vehicles.

The new contributions point to the use of geographic information that allows classifying the population in terms of the type of waste it generates, the requirements of recycling points, density, generation of collection points, among others.

## REFERENCES

- Alinaghian, M., & Shokouhi, N. (2018). Multi-depot multi-compartment vehicle routing problem, solved by a hybrid adaptive large neighborhood search. *Omega*, 76, 85-99. <https://doi.org/10.1016/j.omega.2017.05.002>
- Alweshah, M., Almiani, M., Almansour, N., Al Khalailah, S., Aldabbas, H., Alomoush, W., & Alshareef, A. (2022). Vehicle routing problems based on Harris Hawks optimization. *Journal of Big Data*, 9(1), 42. <https://doi.org/10.1186/s40537-022-00593-4>
- Angelelli, E., & Speranza, M. G. (2002). The Application of a Vehicle Routing Model to a Waste Collection Problem: Two Case Studies. En A. Klose, M. G. Speranza, & L. N. Van Wassenhove (Eds.), *Quantitative Approaches to Distribution Logistics and Supply Chain Management* (pp. 269-286). Springer. [https://doi.org/10.1007/978-3-642-56183-2\\_16](https://doi.org/10.1007/978-3-642-56183-2_16)
- Arnold, F., & Sörensen, K. (2019). What makes a VRP solution good? The generation of problem-specific knowledge for heuristics. *Computers & Operations Research*, 106, 280-288. <https://doi.org/10.1016/j.cor.2018.02.007>
- Ban, H.-B., & Nguyen, P. K. (2021). A hybrid metaheuristic for solving asymmetric distance-constrained vehicle routing problem. *Computational Social Networks*, 8(1), 3. <https://doi.org/10.1186/s40649-020-00084-7>
- Becker, C., Gauthier, J. B., Gschwind, T., & Schneider, M. (2023). In-depth analysis of granular local search for capacitated vehicle routing. *Discrete Applied Mathematics*, 329, 61-86. <https://doi.org/10.1016/j.dam.2022.12.019>
- Beliën, J., De Boeck, L., & Van Ackere, J. (2014). Municipal Solid Waste Collection and Management Problems: A Literature Review. *Transportation Science*, 48(1), 78-102.
- Berhan, E., Beshah, B., Kitaw, D., & Abraham, A. (2014). Stochastic Vehicle Routing Problem: A Literature Survey. *Journal of Information & Knowledge Management*, 13(03), 1450022. <https://doi.org/10.1142/S0219649214500221>
- Bierlaire, M. (2015). Simulation and optimization: A short review. *Transportation Research Part C: Emerging Technologies*, 55(Supplement C), 4-13. <https://doi.org/10.1016/j.trc.2015.01.004>
- Boudali, I., & Ragmoun, M. (2022). A Memetic Approach for Routing Problem with Capacity and Time Constraints. En C. Bădică, J. Treur, D. Benslimane, B. Hnatkowska, & M. Krótkiewicz (Eds.), *Advances in Computational Collective Intelligence* (pp. 612-626). Springer International Publishing. [https://doi.org/10.1007/978-3-031-16210-7\\_50](https://doi.org/10.1007/978-3-031-16210-7_50)
- Braekers, K., Ramaekers, K., & Van Nieuwenhuysse, I. (2016). The vehicle routing problem: State of the art classification and review. *Computers & Industrial Engineering*, 99, 300-313. <https://doi.org/10.1016/j.cie.2015.12.007>
- Brownlee, J. (2012). *Clever Algorithms: Nature-Inspired Programming Recipes*. Lulu.com.
- Chen, J., & Shi, J. (2019). A multi-compartment vehicle routing problem with time windows for urban distribution – A comparison study on particle swarm optimization algorithms. *Computers & Industrial Engineering*, 133, 95-106. <https://doi.org/10.1016/j.cie.2019.05.008>
- Chiussi, A., de Paula Felix, G., Iori, M., & dos Santos, A. G. (2022). Industrial Waste Collection Optimization: A Real-World Case Study in Northern Italy. En J. de Armas, H. Ramalhinho, & S. Voß (Eds.),

Computational Logistics (pp. 147-161). Springer International Publishing. [https://doi.org/10.1007/978-3-031-16579-5\\_11](https://doi.org/10.1007/978-3-031-16579-5_11)

Corberán, Á., Eglese, R., Hasle, G., Plana, I., & Sanchis, J. M. (2021). Arc routing problems: A review of the past, present, and future. *Networks*, 77(1), 88-115. <https://doi.org/10.1002/net.21965>

Deep, L., & Tanmay, S. (2022). Study and Review of Various Techniques on Searching Optimal Route for Online Ordering System. En J. Shah, S. S. Arkatkar, & P. Jadhav (Eds.), *Intelligent Infrastructure in Transportation and Management* (pp. 261-270). Springer. [https://doi.org/10.1007/978-981-16-6936-1\\_21](https://doi.org/10.1007/978-981-16-6936-1_21)

Del Pia, A., & Filippi, C. (2006). A variable neighborhood descent algorithm for a real waste collection problem with mobile depots. *International Transactions in Operational Research*, 13(2), 125-141. <https://doi.org/10.1111/j.1475-3995.2006.00539.x>

Doerner, K. F., Kiefer, A., & Wolfinger, D. (2016). Twenty Years of Vehicle Routing in Vienna. En H. Dawid, K. F. Doerner, G. Feichtinger, P. M. Kort, & A. Seidl (Eds.), *Dynamic Perspectives on Managerial Decision Making: Essays in Honor of Richard F. Hartl* (pp. 491-520). Springer International Publishing. [https://doi.org/10.1007/978-3-319-39120-5\\_26](https://doi.org/10.1007/978-3-319-39120-5_26)

Eglese, R. W., & Letchford, A. N. (2000). Polyhedral Theory for Arc Routing Problems. En M. Dror (Ed.), *Arc Routing: Theory, Solutions and Applications* (pp. 199-230). Springer US. [https://doi.org/10.1007/978-1-4615-4495-1\\_6](https://doi.org/10.1007/978-1-4615-4495-1_6)

El-Gharably, N. E., El-Kilany, K. S., & El-Sayed, A. E. (2013). Optimization Using Simulation of the Vehicle Routing Problem. *International Journal of Industrial and Manufacturing Engineering*, 7(6), 1236-1242.

Elshaer, R., & Awad, H. (2020). A taxonomic review of metaheuristic algorithms for solving the vehicle routing problem and its variants. *Computers & Industrial Engineering*, 140, 106242. <https://doi.org/10.1016/j.cie.2019.106242>

Expósito-Márquez, A., Expósito-Izquierdo, C., Brito-Santana, J., & Moreno-Pérez, J. A. (2018). Solving an Eco-efficient Vehicle Routing Problem for Waste Collection with GRASP. En J. Del Ser, E. Osaba, M. N. Bilbao, J. J. Sanchez-Medina, M. Vecchio, & X.-S. Yang (Eds.), *Intelligent Distributed Computing XII* (pp. 215-224). Springer International Publishing. [https://doi.org/10.1007/978-3-319-99626-4\\_19](https://doi.org/10.1007/978-3-319-99626-4_19)

Fan, W., Xu, H., & Xu, X. (2009). Simulation on vehicle routing problems in logistics distribution. *COMPEL - The international journal for computation and mathematics in electrical and electronic engineering*, 28(6), 1516-1531. <https://doi.org/10.1108/03321640910992056>

Gendreau, M., Laporte, G., & Séguin, R. (1996). Stochastic vehicle routing. *European Journal of Operational Research*, 88(1), 3-12. [https://doi.org/10.1016/0377-2217\(95\)00050-X](https://doi.org/10.1016/0377-2217(95)00050-X)

Ghiani, G., Guerriero, F., Improta, G., & Musmanno, R. (2005). Waste collection in Southern Italy: Solution of a real-life arc routing problem. *International Transactions in Operational Research*, 12(2), 135-144. <https://doi.org/10.1111/j.1475-3995.2005.00494.x>

Gunes, C., van Hoeve, W.-J., & Tayur, S. (2010). Vehicle Routing for Food Rescue Programs: A Comparison of Different Approaches. En A. Lodi, M. Milano, & P. Toth (Eds.), *Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems* (pp. 176-180). Springer. [https://doi.org/10.1007/978-3-642-13520-0\\_21](https://doi.org/10.1007/978-3-642-13520-0_21)

Hasni, S., Toumi, S., Jarboui, B., & Mjirda, A. (2017). GVNS based heuristic for solving the multi-product multi-vehicle inventory routing problem. *Electronic Notes in Discrete Mathematics*, 58, 71-78. <https://doi.org/10.1016/j.endm.2017.03.010>

Herazo-Padilla, N., Montoya-Torres, J. R., Isaza, S. N., & Alvarado-Valencia, J. (2015). Simulation-optimization approach for the stochastic location-routing problem. *Journal of Simulation*, 9(4), 296-311. <https://doi.org/10.1057/jos.2015.15>

Juan, A. A., Faulin, J., Pérez-Bernabeu, E., & Domínguez, O. (2013). Simulation-Optimization Methods in Vehicle Routing Problems: A Literature Review and an Example. En *Modeling and Simulation in Engineering, Economics, and Management* (pp. 115-124). Springer, Berlin, Heidelberg. [https://doi.org/10.1007/978-3-642-38279-6\\_13](https://doi.org/10.1007/978-3-642-38279-6_13)

Kocat&uuml, F., rk, T&uuml, G. Y., t&uuml, nc&uuml, & Salhi, S. (2021). The multi-depot heterogeneous VRP with backhauls: Formulation and a hybrid VNS with GRAMPS meta-heuristic approach. *Annals of Operations Research*, 307(1-2), 277-303.

Laporte, G. (2009). Fifty Years of Vehicle Routing. *Transportation Science*, 43(4), 408-416. JSTOR.

Lesch, V., König, M., Kounev, S., Stein, A., & Krupitzer, C. (2022). Tackling the rich vehicle routing problem with nature-inspired algorithms. *Applied Intelligence*, 52(8), 9476-9500. <https://doi.org/10.1007/s10489-021-03035-5>

Li, J., Cai, L., Lichen, J., Pan, P., Wang, W., & Liu, S. (2021). An Approximation Algorithm for Solving the Heterogeneous Chinese Postman Problem. *Algorithmic Aspects in Information and Management*, 38-46. [https://doi.org/10.1007/978-3-030-93176-6\\_4](https://doi.org/10.1007/978-3-030-93176-6_4)

Mahéo, A., Rossit, D. G., & Kilby, P. (2022). Solving the integrated bin allocation and collection routing problem for municipal solid waste: A Benders decomposition approach. *Annals of Operations Research*. <https://doi.org/10.1007/s10479-022-04918-7>

Máximo, V. R., & Nascimento, M. C. V. (2021). A hybrid adaptive iterated local search with diversification control to the capacitated vehicle routing problem. *European Journal of Operational Research*, 294(3), 1108-1119. <https://doi.org/10.1016/j.ejor.2021.02.024>

Minh, T. T., Van Hoai, T., & Nguyet, T. T. N. (2013). A Memetic Algorithm for Waste Collection Vehicle Routing Problem with Time Windows and Conflicts. En B. Murgante, S. Misra, M. Carlini, C. M. Torre, H.-Q. Nguyen, D. Tanar, B. O. Apduhan, & O. Gervasi (Eds.), *Computational Science and Its Applications – ICCSA 2013* (pp. 485-499). Springer. [https://doi.org/10.1007/978-3-642-39637-3\\_38](https://doi.org/10.1007/978-3-642-39637-3_38)

Mohammed, M. A., Ghani, M. K. A., Hamed, R. I., Mostafa, S. A., Ahmad, M. S., & Ibrahim, D. A. (2017). Solving Vehicle Routing Problem by Using Improved Genetic Algorithm for Optimal Solution. *Journal of Computational Science*. <https://doi.org/10.1016/j.jocs.2017.04.003>

Mohammed, M. A., Mohd Khanapi Abd Ghani, Obaid, O. I., Mostafa, S. A., Ahmad, M. S., Dheyaa Ahmed Ibrahim, & Burhanuddin, M. A. (2017). A Review of Genetic Algorithm Applications in Solving Vehicle Routing Problem. <https://www.medwelljournals.com/abstract/?doi=jeasci.2017.4267.4283>

Parragh, S. N., Doerner, K. F., & Hartl, R. F. (2008a). A survey on pickup and delivery problems. *Journal Für Betriebswirtschaft*, 58(2), 81-117. <https://doi.org/10.1007/s11301-008-0036-4>

Parragh, S. N., Doerner, K. F., & Hartl, R. F. (2008b). A survey on pickup and delivery problems. *Journal Für Betriebswirtschaft*, 58(1), 21-51. <https://doi.org/10.1007/s11301-008-0033-7>

Policroniades Chípuli, G., & Flores de la Mota, I. (2021). Analysis, design and reconstruction of a VRP model in a collapsed distribution network using simulation and optimization. *Case Studies on Transport Policy*. <https://doi.org/10.1016/j.cstp.2021.07.002>

Policroniades, G., & Flores de la Mota, I. (2016). State of the Art of the Different Models of Transportation Most Used in the Supply Chain of Automotive Industry. *International Journal of Combinatorial Optimization Problems and Informatics*, 7(3), 44-53.

Policroniades, G., Flores de la Mota, I., Sibirai Reyna, O. S., & Lara de Paz, J. (2018). The Vehicle Routing Problem Complex Network Analysis. *Journal of Applied Mathematics and Computation*, 2(12). <https://doi.org/10.26855/jamc.2018.12.001>

Queiroga, E., Frota, Y., Sadykov, R., Subramanian, A., Uchoa, E., & Vidal, T. (2020). On the exact solution of vehicle routing problems with backhauls. *European Journal of Operational Research*, 287(1), 76-89. <https://doi.org/10.1016/j.ejor.2020.04.047>

Quila, D., Morillo, D., Cabrera, G., Linfati, R., & Gatica, G. (2020). A New Mathematical Model for the Vehicle Routing Problem with Backhauls and Time Windows. En Á. Rocha, C. Ferrás, C. E. Montenegro Marin, & V. H. Medina García (Eds.), *Information Technology and Systems* (pp. 46-53). Springer International Publishing. [https://doi.org/10.1007/978-3-030-40690-5\\_5](https://doi.org/10.1007/978-3-030-40690-5_5)

Raffaele, A., Zattereri, M., Bazzoli, F., Gussago, M., & Rizzi, R. (2023). A Two-Phase Approach to Evaluate and Optimize an Interlibrary Loan Service: The Case Study of Provincia di Brescia. En M. Cosmi, L. Peirano, A. Raffaele, & M. Samà (Eds.), *Operations Research and Data Science in Public Services* (pp. 17-33). Springer International Publishing. [https://doi.org/10.1007/978-3-031-34546-3\\_2](https://doi.org/10.1007/978-3-031-34546-3_2)

Rezaei, B., Guimaraes, F. G., Enayatifar, R., & Haddow, P. C. (2023). Combining genetic local search into a multi-population Imperialist Competitive Algorithm for the Capacitated Vehicle Routing Problem. *Applied Soft Computing*, 142, 110309. <https://doi.org/10.1016/j.asoc.2023.110309>

Rodrigues, A. M., & Soeiro Ferreira, J. (2015). Waste collection routing—Limited multiple landfills and heterogeneous fleet. *Networks*, 65(2), 155-165. <https://doi.org/10.1002/net.21597>

Rodriguez, H., Díaz, J., Fajardo-Calderín, J., Perallos, A., & Onieva, E. (2021). Waste Collection Vehicle Routing Problem Using Filling Predictions. En H. Sanjurjo González, I. Pastor López, P. García Bringas, H. Quintián, & E. Corchado (Eds.), *Hybrid Artificial Intelligent Systems* (pp. 637-648). Springer International Publishing. [https://doi.org/10.1007/978-3-030-86271-8\\_53](https://doi.org/10.1007/978-3-030-86271-8_53)

Salehi Sarbijan, M., & Behnamian, J. (2022). Multi-fleet feeder vehicle routing problem using hybrid metaheuristic. *Computers & Operations Research*, 141, 105696. <https://doi.org/10.1016/j.cor.2022.105696>

Santos, M. J., Jorge, D., Ramos, T., & Barbosa-Póvoa, A. (2023). Green reverse logistics: Exploring the vehicle routing problem with deliveries and pickups. *Omega*, 118, 102864. <https://doi.org/10.1016/j.omega.2023.102864>

Segura, E. L., Melquiades, J. R., & Segura, F. G. (2022). Tabu Search for Locating-Routing in the Goods Delivery and Waste Pickup in Trujillo-Peru. En K. Arai (Ed.), *Intelligent Systems and Applications* (pp. 51-69). Springer International Publishing. [https://doi.org/10.1007/978-3-030-82196-8\\_5](https://doi.org/10.1007/978-3-030-82196-8_5)

Sgarro, G. A., Santoro, D., & Grilli, L. (2024). Ant Colony Optimization for solving Directed Chinese Postman Problem. *Neural Computing and Applications*, 36(28), 17615-17630. <https://doi.org/10.1007/s00521-024-10052-1>

Simsir, F., & Ekmekci, D. (2019). A metaheuristic solution approach to capacitated vehicle routing and network optimization. *Engineering Science and Technology, an International Journal*, 22(3), 727-735. <https://doi.org/10.1016/j.jestch.2019.01.002>

Solano-Charris, E., Prins, C., & Santos, A. C. (2015). Local search based metaheuristics for the robust vehicle routing problem with discrete scenarios. *Applied Soft Computing*, 32, 518-531. <https://doi.org/10.1016/j.asoc.2015.03.058>

Solomon, M. M. (1987). Algorithms for the Vehicle Routing and Scheduling Problems with Time Window Constraints. *Operations Research*, 35(2), 254-265.

Subramanian, A., & Queiroga, E. (2020). Solution strategies for the vehicle routing problem with backhauls. *Optimization Letters*, 14(8), 2429-2441. <https://doi.org/10.1007/s11590-020-01564-5>

Sukhpal, & Kumar, K. (2023). Multi-trip multi-compartment vehicle routing problem with backhauls. *International Journal of System Assurance Engineering and Management*. <https://doi.org/10.1007/s13198-023-02040-w>

Tlili, T., Faiz, S., & Krichen, S. (2014). A Hybrid Metaheuristic for the Distance-constrained Capacitated Vehicle Routing Problem. *Procedia - Social and Behavioral Sciences*, 109, 779-783. <https://doi.org/10.1016/j.sbspro.2013.12.543>

Torres Matus, R. (2005). Un Algoritmo Genético para Resolver el Problema de Asignar y Secuenciar Entregas con Restricción de Ventanas de Tiempo-Edición Única—Repositorio del Tecnológico de Monterrey. <https://repositorio.itesm.mx/ortec/handle/11285/572509>

Vaferi, M., Fallah, M. S., & Tayebi, A. H. (2018). A Metaheuristic for the Containership Feeder Routing Problem with Port Choice Process. *The Asian Journal of Shipping and Logistics*, 34(2), 119-128. <https://doi.org/10.1016/j.ajsl.2018.06.008>

Valizadeh, J., Mozafari, P., & Hafezalkotob, A. (2021). Municipal waste management and electrical energy generation from solid waste: A mathematical programming approach. *Journal of Modelling in Management*, 17(1), 309-340. <https://doi.org/10.1108/JM2-07-2020-0193>

Vieira, B. S., Ribeiro, G. M., & Bahiense, L. (2023). Metaheuristics with variable diversity control and neighborhood search for the Heterogeneous Site-Dependent Multi-depot Multi-trip Periodic Vehicle Routing Problem. *Computers & Operations Research*, 153, 106189. <https://doi.org/10.1016/j.cor.2023.106189>

Voigt, S., Frank, M., Fontaine, P., & Kuhn, H. (2022). Hybrid adaptive large neighborhood search for vehicle routing problems with depot location decisions. *Computers & Operations Research*, 146, 105856. <https://doi.org/10.1016/j.cor.2022.105856>

Wang, Y., Assogba, K., Fan, J., Xu, M., Liu, Y., & Wang, H. (2019). Multi-depot green vehicle routing problem with shared transportation resource: Integration of time-dependent speed and piecewise penalty cost. *Journal of Cleaner Production*, 232, 12-29. <https://doi.org/10.1016/j.jclepro.2019.05.344>

Wu, H., Gao, Y., Wang, W., & Zhang, Z. (2023). A hybrid ant colony algorithm based on multiple strategies for the vehicle routing problem with time windows. *Complex & Intelligent Systems*, 9(3), 2491-2508. <https://doi.org/10.1007/s40747-021-00401-1>

Xin, J., Yu, B., D'Ariano, A., Wang, H., & Wang, M. (2022). Time-dependent rural postman problem: Time-space network formulation and genetic algorithm. *Operational Research*, 22(3), 2943-2972. <https://doi.org/10.1007/s12351-021-00639-0>

Zenil Chávez, H. (2005). Computación evolutiva y sistemas complejos adaptativos: La sopa primigenia de las conductas inteligentes emergentes. *Ciencia - Academia Mexicana de Ciencias*, 56(2), Article 2.